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An Efficient Dynamic Approach for Exact Reconstruction for CT Image Application

S.Asif Hussain, P.Syamala devi, C.Hemanth

Department of ECE, A.I.T.S,
Rajampet, Andhra Pradesh, India

Abstract— **For reconstructing volume of filtered projection data a parallel computing algorithm is proposed which suits spiral cone beam CT access for improvement of speed with high end processor. This work uses phantom data to achieve higher speed by accessing memory many times here the work is carried frame by frame as the pixel data arrives from filtered back projection; Interpolation is used for dedicated volume data which is decided by the coordinates of neighborhood pixels. Image quality is achieved in real time to refine artifacts by using parallel computing algorithm, experiment results shows the reduction of memory accessing times from many to one.**

Keywords- Filtered back projection, Parallel computing Spiral cone beam CT, Artifacts

I INTRODUCTION

Digital image reconstruction is a robust by means by which the underlying images hidden in blurry and noisy data can be reveled [14]. The main challenge is sensitivity to measure noise to the input data, which can be magnified strongly, resulting in large artifacts in the reconstructed image .Many imaging techniques are based on reconstructing an image from data that can be interpreted, either directly or after some preprocessing, as a set of projections of the imaged object. The mathematical foundation is provided by the Radon transform (RT) which computes 1-D projections of a 2-D data at different view angles. Different from X-ray radiograph, the inner structure of the FOV can be detected with CT, or the CT image has much better space resolution than tradition X-ray radiograph [14]. CT has gain widely use as a tool not only in medical as medical imaging, noninvasive diagnostics and surgical planning but also in industry for nondestructive inspection[15]. In CT imaging, for example, the data is obtained by passing a set of narrow X-ray beams through the scanned object and collecting their intensities using an array of sensors. The acquired data represents the Radon transform of the cross-sectional absorption densities that form the image [1]. Munson et al. [2] showed that the data collected by the SAR, after demodulation and low pass filtering, represents the Fourier transform of the projections obtained from the reflectivity density of the targeted ground patch.

X-ray computed tomography is a technique closely combined with the X-ray radiation. With the technique, the inner structure of an N dimensional object can be reconstructed from the N -1 dimensional projections [4] Different from X-ray radiograph, the inner structure of the FOV can be detected with CT, or the CT image has much better space resolution than tradition X-ray radiograph[12].

However most popular in practice are methods based on the back projection (BP) operation which reduces the distortion by avoiding the interpolation step. The approach is also more suitable to handle other problems such as wave front curvature effects in SAR imaging the image reconstruction algorithm for the spiral cone beam CT can be divided into two categories, the approximate and the exact. Typically, the former one contains various kinds of the FDK-type algorithm transformed from the classic FDK-algorithm for circular cone beam CT which is widely used in the clinic now a days . The advantage of the FDK algorithm includes its fast computing speed and good image quality. But the disadvantage is also obvious, that is the cone beam angle could not be too big, or the artifact becomes very serious. Nowadays, the spiral cone-beam CT is not only the mainstream for its really fast scanning speed and the ability to produce truly 3D image and [5,6]. In 2002, katsevich proposed the first theoretical exact reconstruction formula for spiral; cone beam CT [7,8]. It is of truly FBP type with one dimensional shift invariant filtering, as a result the computation is more efficient than the random transform based reconstruction algorithm .It has feature of solving the long object problem and uses the data inside the Tam-Danielson window[9] and some more outside the window. Though it's numerous advantages, the bottle neck is still obvious for its intensive computation. After the formula came out, several numerical study and implementation for the formula has been reported

In 2007, Jiang proposed a fast algorithm for katsevichs formula, the cone beam cover method [10] different from the pi –line method; the new method adopts the new concept cone beam cover. The method can update the voxel defined in the cone beam cover for each projection frame, so a parallel computing can be achieved .then, and then implement the new method with a Linux cluster [12] .Each frame occupies a computation node

In this paper, we focus the back projection (BP) operation the computational bottle neck [13] of the FBP algorithm.

II . BACKGROUND

An essential step in image reconstruction is back projection, which is the ad joint to forward Projection process that forms the projections of the object. Figure 1 shows the back projection along a fixed angle, Ø Conceptually, back projection can be described as placing a value of p(s,Ø) back into an image array along the appropriate LOR, but, since the knowledge of where the values came from was lost in the projection step, the best we can do is place a constant value into all elements along the LOR.

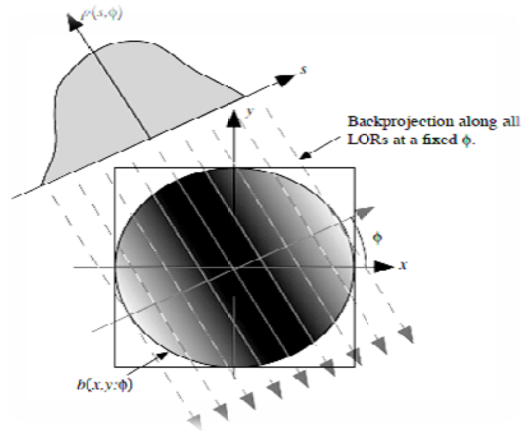

Fig 1. Back projection b(x,y,Ø), into an image reconstruction array of all values P(S,Ø) for a fixed value of Ø

One might assume that straight back projection of all the collected projections will return the Image, but this is not the case due to the oversampling in the center of the Fourier transform. In Other words, each projection fills in one slice of the Fourier space resulting in oversampling in the center and less sampling at the edges. For example, if we perform back projections at only two angles, say Ø1 and Ø2 and examine the Fourier transform of the result we see that the contribution at the origin is doubled while there is only one contribution at the edges of the field of view. Another way of understanding this oversampling in the space domain is with the forward projection of a single point source. If we simply back project the point source projections, the image would be heavily blurred since the projections are added back to the entire LOR from which they came. The oversampling needs to be re-weighted, or 'filtered', in order to have equal contributions throughout the field of view

A. filtered-back projection (fbp) reconstruction

Our goal is to compute f (x, y) from p(s, Ø) . After back projection, the oversampling in the center of Fourier space needs to be filtered in order to have equal sampling throughout the Fourier space. Basically, the Fourier transform of the back projected image must be filtered with a 'cone' filter

υ= $\sqrt{v_x^2+v_y^2}$ this cone filter accentuates values at the edge of the Fourier space and de accentuates values at the center of the Fourier space. This operation is summarized in

$$F\,(v_x, v_y) = vB(v_x, v_y)$$

Where B (vx, vy) is the 2-d Fourier transform of the back projected image and F(v_x, v_y) is the 2-D Fourier transform of the back projection-filtered image. The final step is the inverse Fourier transform of F(v_x, v_y) to obtain the image F(x,y) This is known as the back projection-filtering (BPF) image reconstruction method, where the projection data are first back projected, filtered in Fourier space with the cone filter, and then inverse Fourier transformed. Alternatively the filtering can be performed in image space via the convolution of b(x, y) with $F_2^{-1}\{v\}$ A disadvantage of this approach is that the function $b(x, y)$ has a larger support than $f\,(x, y)$ due to the convolution with the filter term, which results in gradually decaying values outside the support of $f\,(x,\ y)$. Thus any numerical procedure must first compute $b(x,\ y)$ using a significantly larger image matrix size than is needed for the final result. This disadvantage can be avoided by interchanging the filtering and back projection steps as discussed next.

B. Reconstruction by filtered-back projection (FBP)

If we interchange the order of the filtering and back projection steps ,we obtain the Useful filtered-back projection (FBP) image reconstruction method:

$$F(x, y) = \int_0^{\pi} p^F\,(s, \emptyset) d\emptyset$$

Where the 'filtered' projection, given by

$$P^F(S, \emptyset) = F_1^{-1}\{|v_S|\; F_1\{P(S,\emptyset)\}\}$$

Can be regarded as pre-corrected for the oversampling of the Fourier transform of f(x,y). The one dimensional 'ramp' filter $|vs|$ is a section through the rotationally symmetric two-dimensional cone filter. An advantage of FBP is that the ramp filter is applied to each measured projection, which has a finite support in s, and we only need to back project the filtered projections for $|s|$ less than the radius of the field of view. This means that with FBP the image can be efficiently calculated with a much smaller reconstruction matrix than can be used with BPF, for the same level of accuracy. This is part of the reason for the popularity of the FBP algorithm

III. METHODS

A. FILTERED BACKPROJECTION ALGORITHMS

The Radon transform (RT) represents a set of parallel line integral projections of a 2-D function f(x, y) at different angles θ. The continuous Radon transform is defined by

$$F(r, \theta) = \iint f\,(x, y)\delta(r - xcos\theta - ysin\theta)\,dx\,dy$$

Where r and θ are polar coordinates and δ is the unit impulse. The projections ˆ f(r, θ) are also referred to as the data as Sinograms. In their original form, filtered back projection (FBP) algorithms are

Based on the well-known inversion formula for the RT:

f(x,y)=$\beta\vec{f}(p, \theta)$,

$$\bar{f}\,(\rho, \theta) = F_R^{-1}\,|\omega_r|\,F_r\,\hat{\bar{f}}\,(r, \theta).$$

Here $\widehat{F}\,(\omega_{r,}\,\theta) = F_r\hat{f}(r, \theta)$ represents a 1-D Fourier transform in the variable r, and B is the continuous back projection operator

$$F(x,y) = \beta\,\hat{f}(\rho, \theta) = \int_0^{\pi} \hat{f}\,(xcos\theta + ysin\theta)\,d\theta$$

The projections f(r, θ) are first filtered using the ramp filter |ωr| and then back projected to reconstruct the image.

B. DISCRETE DIRECT BACK PROJECTION.

In practice, the number of projections P and the sampling distribution are determined by the data acquiring equipment, and the reconstructed image is discrete. We will assume that the projection angles θ are evenly distributed in the interval [0, π) and that all images are square with N × N pixels. To implement the FBP algorithm on a computer, the back projection operation is discretized and the ramp filter is windowed and sampled. The discrete back projection is performed for each pixel f(m, n) as a sum of projected values over all angles θ:

$F(m, n) = \sum \vec{f}(m\cos\theta + n\sin\theta, \theta)$.

Interpolation with a kernel φ (ρ) in the radial direction is required to compute sampled $\overleftarrow{f}$ at non-integral values. Better approximation to the continuous back projection can be achieved by introducing the image sampling operator to model the physical properties of the sensing equipment [3]. In our implementation, however, we use the ideal sampling kernel.

IV. PRACTICAL RECONSTRUCTION APPROACH
A. THE SPIRAL CONE-BEAM CT GEOMETRY

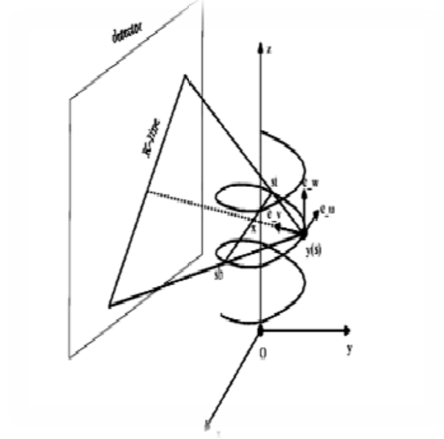

Figure 1. The geometry of spiral cone beam CT

The geometry of the spiral cone-beam CT is shown in Fig. 1. Here, $\overleftarrow{x}$ is the voxel to be reconstructed. In the Cartesian coordinate system O-XYZ. $\overleftarrow{Y}$ Denotes the X-ray source position which can be denoted as

$$\vec{Y}(S) = [R\cos(s) \,.\, R\sin(s)\,, \frac{\pi}{2\pi}s]^T$$

Where s is the rotation angel of the source and R is the radius of the support cylinder.

Then, the unit vector pointing from $\vec{Y}$(S) to $\vec{x}$ is :

$$\hat{\beta}(\vec{x}, s) = \frac{\vec{x} - \vec{y}(s)}{\|\vec{x} - \vec{y}(s)\|}$$

Now, a rotating coordinate is adopted at source position $y(s)$ with its two axis's $\hat{e}_u$ and $\hat{e_v}$ parallel

To the detector plane while the axis ˆ*w e* perpendicular to the detector plane, defined by:

$$\hat{e}_u = [-\sin(s)\,, \cos(s)\,, 0]^T$$

$$\hat{e}_v = [-\cos(s)\,, -\sin(s)\,, 0]^T$$

$$\hat{e}_w = [0,0,1]^T$$

On the detector plane, a third coordinate is defined with origin at the projection of the X-ray source on the detector plane and the two axis's parallel to .$\hat{e}_u$ and $\hat{e_v}$ In this way, the detector plane is expanded by the two vectors.

B. Proposed algorithm:

Parallel back projection

Let's make the following definition

$$FP_S = \int_0^{2\pi} \frac{\partial}{\partial q} \mathrm{Df}(\vec{Y}(\mathrm{s}), \vec{\theta}(\vec{x}s, \gamma))d\gamma$$

Obviously, *s FP* is the filtered projection data at source position *s* . Furthermore, let's make the

Following definition:

$$\delta\mathrm{f}(\vec{x})_s = \frac{1}{\|\vec{x} - \vec{y}(s)\|}\mathrm{F}P_S$$

Then,

$\mathrm{F}(\vec{x}) =$

$$\frac{1}{2\pi}\int_{I_{PI_x}} \frac{1}{\|\vec{X} - \vec{Y}(S)\|} FP_S ds = \frac{1}{2\pi}(\delta f(\vec{x})_{s_b} + \ldots\ldots\ldots \delta\mathrm{f}(\vec{x})_{s_t})$$

Here is the π- line decided by the voxel $\vec{x}$.

we can see that for each source positions s on the there exists a $\delta\mathrm{f}(\vec{x})$. we say that

$\mathrm{f}(\vec{x})$ is updated at source positions s by $\delta\mathrm{f}(\vec{x})_s$.

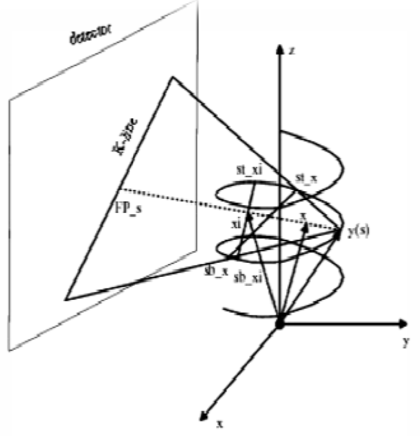

Figure 3. The geometry for the new algorithm

As shown in Fig. 3., let $\vec{x}_i$ to be the voxels on the ray connecting source s and voxel $\vec{x}_i$. Obviously, the following equation holds:

$$\delta\mathrm{f}(\vec{x}_i)_s = \frac{1}{\|\vec{x}_i - \vec{y}(s)\|} FP_S$$

According to the sufficient condition of π -line, we can make the projection not at the source on the π Parametric interval to be 0.Then, the following equation holds

$$\delta\mathrm{f}(\vec{x}_i)s = \begin{cases} \frac{1}{\|\vec{x} - \overline{\vec{y}(s)}\|} \quad FP_S, & \text{if } \mathrm{s}\, \epsilon I_{PI_{xi}} \\ 0 & \text{else} \end{cases}$$

This indicates that if the projection data of a ray is given, then all the voxels on the ray can be updated.
Here, we define the voxels on a ray to be:

$X=\{\overrightarrow{X_I} \mid \overrightarrow{X_I} \epsilon Ray_s^p, s\epsilon I_{PI_{X1}}, P \in FP\}$

Where Ray_s^p connects sourse s and projection FP_S . so X is the set of the voxels on Ray_s^p
Now

$\Delta f(x)=\frac{1}{\|x-\vec{y}(s)\|} FP_S$

Then the exact reconstruction formula can be expressed as:

$f(X) = \frac{1}{2\pi}(\delta f(x)_{s_b} + \ldots\ldots \delta f(x)_{s_r}) = \frac{1}{2\pi}\int_{IPIZ} \frac{1}{\|X-\vec{Y}(S)\|} FP_S d_s$

Eqn.updates the voxels on the ray connecting source *s* and projection FP_S If we define *FP* to be the set of all the projection at source s , *s Ray* to be all the ray pass $Ray_{S.}X_S$ to be all the voxels on *s Ray* . Then we come to the parallel reconstruction formula:

$F(X_S) = \frac{1}{2\pi}(\delta f(X_S)_{S_B} + \ldots\ldots \delta f(X_S)_{S_I} =$

$\frac{1}{2\pi}\int I_{PI_{X_I}} \frac{1}{\|X_Z-\vec{Y}(S)\|} FP\, ds$

We can see from the equation that the voxels in *s X* can be updated if one frame of filtered projection arrives. When the next frame of filtered projection arrives, another set of *s X* can be updated.
The voxel to be updated has the following features:
(1) The voxel is in the support cylinder
(2) the source *s* is on the π parametic interval decided by voxel

V. THE EXACT RECONSTRUCTION FORMULA

The exact formula proposed by Katsevich can be expressed as following:

$f(\vec{x})=-\frac{1}{2\pi}\int I_{PI} \frac{1}{\|\vec{X}-\vec{Y}(S)\|}\int_0^{2\pi} \frac{\partial}{\partial Q} Df(\vec{Y}(S),\vec{\theta})) \mid_{Q-S} \frac{1}{\sin\gamma} d\gamma$

Here $Df(\vec{y},\vec{\theta})$ is the projection of the reconstructed object . I_{PI} is the π parametric interval.
From the equation we can see that the projection data is first filtered, then the filtered projection data is Back projected to form the attenuation image. It's of truly FBP type.
The formula can be solved by the following five steps:
(1) Derive the projection data Df(y,θ) using chain rule

$$g_1(s,u,w)\frac{\partial}{\partial q} Df\left(\vec{y}_q, \vec{\theta}(s,x,\gamma)\right) \mid_{q-z}$$

$$\frac{1}{sin\gamma} d\gamma \frac{\partial_{gf}}{\partial_q} \frac{u^2+D^2}{D} \frac{\partial_{gf}}{\partial u} \frac{uv}{D} \frac{\partial gf}{\partial_W} \mid_{q-Z}$$

(2) Length weight correction

$g_2(s,u,w) = \frac{D}{\sqrt{U^2+D^2+W^2}} g_1(s,u,w)$

(3) Rebinning

Define r to be the maximum[15] radius of the object α_m = arcsin(r/R), using linear interpolation to all the

$\Psi_n \epsilon$ [$-\pi/2-\alpha_{m,} \frac{\pi}{2} + \alpha_m$]

$$g_3(s,u,\Psi) = g_2(s,u,w_k(u,\Psi))$$

Here $w_k(u,\Psi)=\frac{DH}{R}(\Psi+\frac{\Psi}{\tan\Psi}\frac{u}{D})$

(4) IDfiltering

$g_4(s,u,\Psi)=\int_{-\pi}^{\pi} k_H(u-v)\, g_3(s,u,w_k(u,\Psi)du$

(5) Rebinning

$g_5(s,u,w) = g_4\left(s,u,\vec{\Psi}(u,w)\right)$

Here $\vec{\Psi}(u,w)$ is th one with the least resolution value , satisfyig

$w_k = \frac{D_h}{R}(\Psi + \frac{\Psi}{\tan\Psi}\frac{u}{D})$.

(6) BACK PROJECTION

$$f(\vec{x}) = \frac{1}{2\pi}\int_{S_t}^{S_b} \frac{g_5(s,u^*,w^*)}{v^*(s,x)} ds$$

VI. SIMULATION RESULTS

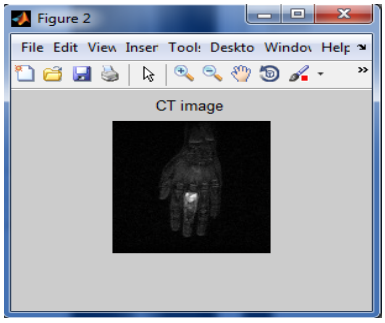

Fig1:Shows the CT image

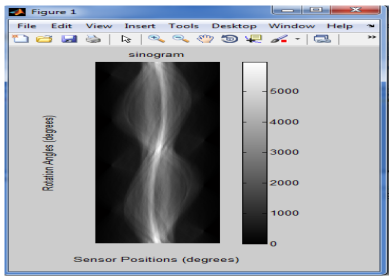

Fig 2:The sinogram image acquired from CT

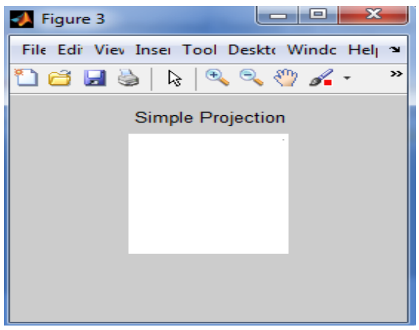

Fig 3 : The simple projection image

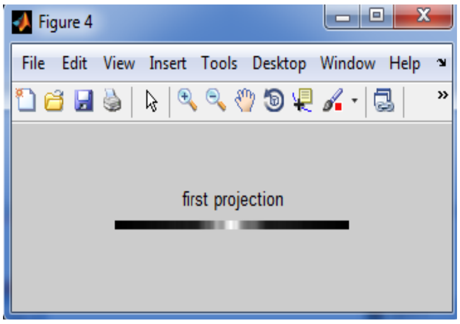

Fig 4 : The first projection on the sonogram

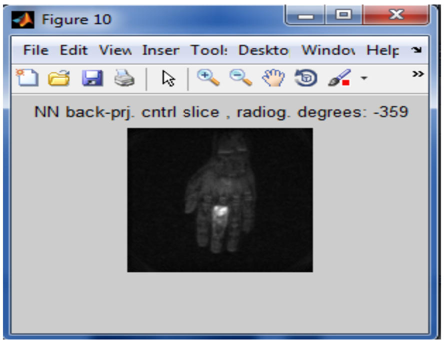

Fig 5: The construction of back projection image

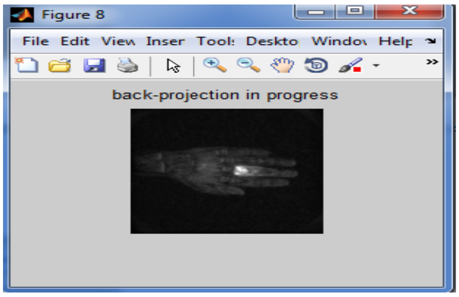

Fig 6: The back projection in action

VII. CONCLUSION

The implementation of the exact reconstruction algorithm for spiral cone-beam CT is quite time consuming for its intensive computation. Among them, the frequent access of the memory is quite a bottle neck to improve the performance. In this paper, we propose a parallel back projection algorithm that can reduce the memory accessing times from many times down to once. As some certain attenuation value can be updated once a filtered back projection data arrives, a pipeline can be achieved for the reconstruction. From the experiment results, we can see that the image quality reconstructed by the new algorithm is satisfactory. The new method may provide some suggestions to engineers or researchers intending to implement the exact reconstruction algorithms in hardware. Though, there are some further works to be done. The most important one is to develop some more interpolation methods for the filtered projection data, and find some new parallel algorithm as the artifact can be greatly reduced if proper interpolation can be employed.

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AUTHORS PROFILE

Chennakesavula Hemanth received B.Tech Degree in Electronics.& Communication Engg. from J.N.T.University, Ananthapur, India. Presently he is with Annamacharya Institute of Technology & Sciences, Rajampet, A.P., India in Dept. of ECE and pursuing his M.Tech. His research interests include Signal Processing, Time Series Analysis and Image Processing.

Asif Hussian. Shaik received B.Tech & M.Tech Degree in Electronics & Communication Engg. from JNT University, Hyderabad, India. He is currently working towards PhD Degree in Biomedical Image Processing at JNTU University, Anantapur, India. Presently he is with Annamacharya Institute of Technology & Sciences, Rajampet, A.P., India. He is working as Assistant Professor in Dept. of ECE. He presented many research papers in National & International Conferences & journals. He is a member of Professional societies like ISTE (India), BMESI (India), IACSIT (Singapore), IAENG (Hongkong) and WASE (Hongkong).His research interests include Signal Processing, Time Series Analysis and Image Processing.